Rotation of galaxies as a signature of cosmic strings in weak lensing surveys

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Vector perturbations sourced by topological defects can generate rotations in the lensing of background galaxies. This is a potential smoking gun for the existence of defects since rotation generates a curl-like component in the weak lensing signal which is not generated by standard density perturbations at linear order. This rotation signal is calculated as generated by cosmic strings. Future large scale weak lensing surveys should be able to detect this signal even for string tensions an order of magnitude lower than current constraints.

Introduction. Weak lensing of background galaxies has earned a place in the growing observational toolkit of the era of precision cosmology. The shearing of galaxies out to redshifts of a few has become a routine measurement [1]. These measurements hold much promise in the quest to constrain cosmological parameters with a particular focus on the equation of state of dark energy [2, 3]. The conventional picture first proposed in [4] is one where correlations in the weak lensing of photon bundles can be related statistically to the power spectrum of the evolving density field along the line of sight. The correlations can be observed by measuring the shearing of background objects such as galaxies or higher redshift objects such as Ly-α emitters or CMB anisotropies.

The statistical effect of the weak lensing is well understood. Scalar perturbations such as density fluctuations generate three independent components of the matrix relating the original source to the distorted image. The first is a trace κ which gives the amplification, or convergence of the image and the second are two shear components, γ_1 and γ_2, which describe a symmetric, traceless, and divergenceless contribution to the distortion matrix. A fourth independent component ρ, describing rotations, can be added as an anti-symmetric contribution. However, ρ cannot be generated by linear perturbations transforming as scalars under 3d rotations. In fact any rotational, or curl-like, component in the surveys has been used as measure of systematic contamination of the data [5].

A number of authors [6–8] have extended the formalism to account for the fact that scalar density perturbations can source ρ via generation of vector (bulk flows) and tensor (gravity waves) at second order. In both cases however, the signal is expected to be very small and it will be a significant challenge to measure even with future surveys. Another source is intrinsic correlations in galaxies [9, 10].

This *letter* suggests an alternative source of curl-like distortions at first order in the perturbation amplitude. The source of the signal are vector metric perturbations induced by cosmic strings along the line of sight. Cosmic strings were first predicted in the context of symmetry breaking phase transitions in the early universe [11]. They arise as topological defects along lines where a complex field has remained trapped in a false vacuum after a symmetry breaking phase transition where the field rolls down to a global vacuum selecting a random phase.

For many years cosmic strings provided an "active" alternative to the "passive" structure formation scenarios based on inflationary generated passive perturbations (the terminology originates in [12]). The passive picture became accepted as the predominant mechanism when the first acoustics peaks, a clear prediction of the coherent passive scenario, were detected in the CMB [13, 14]. However a sub-dominant contribution from cosmic strings has not been ruled out [15, 16]. In recent years renewed interest in cosmic strings has also been driven by the possibility that many string theory models predict the generation of macroscopic strings at the end of inflation [17].

Cosmic strings carry energy and momentum and source perturbations to the metric. The metric perturbations in turn lead to lensing of photon trajectories close to the strings. This is the source of the well known Kaiser–Stebbins effect [18, 19] where a moving string causes a line–like discontinuity in the CMB temperature. The signal induced in cosmic shear surveys by the scalar source of a network of strings is smaller than that due to dark matter density perturbations along the line of sight given current constraints on string tensions $G\mu < 0.7 \times 10^{-6}$ (for the Abelian model [15]). In contrast, the signal due to vector and tensor perturbations sourced by strings generates rotations which have no counterpart, at the same order of magnitude, from density perturbations. Thus any observations of curl–like lensing signal would provide a candidate detection of cosmic strings.

This *letter* focuses on the vector mode induced signal which is expected to be an order of magnitude greater than the tensor induced one [20]. The *letter* is organized as follows; the lensing distortion generated by a vector source is first calculated and then applied to the case of a single, moving, straight string. Finally the statistical signal due to a network of strings is computed and compared with the expected variance of future weak lensing surveys. Throughout units where $c = 1$ are used unless otherwise stated. Greek indices run over all spacetime dimensions with latin indices running only over the spatial dimensions. Overdots denote differentiation with respect to conformal time η and a $(-+++)$ signature is adopted for the metric.

Vector sourced distortions. Generalised, vector–type perturbations to the flat Friedmann-Robertson-Walker metric are given by the contributions $g_{0i} = -a^2 V_i$ and

$g_{ij} = a^2(F_{i,j} + F_{j,i})$, with both F_i and V_i are divergenceless vectors and $a(\eta)$ is the scale factor. Two of the four independent modes specified by the two vectors can be fixed by a choice of gauge and $F_i = 0$ is adopted for this calculation. The geodesic equation can then be used to derive the effect of the perturbed metric on the trajectory of photons [21]. The coordinates can be aligned such that $x^i = (x, y, z) \equiv \chi(\theta_1, \theta_2, 1)$ where χ is the comoving radial distance with $d\chi/d\eta = 1$ and $\vec{\theta}$ is the vector spanning the plane orthogonal to the line of sight. Using the relation $d\eta/d\lambda = p/a$, where p is the modulus of the photon 3-momentum, a second order differential equation for the transverse projection of the trajectory is obtained

$$\frac{d^2(\chi\theta_i)}{d\chi^2} = \frac{\dot{V}_i}{a^2} + \frac{V_{z,i}}{a^2} - \frac{V_{i,z}}{a^2}, \tag{1}$$

where $i = 1$ and 2 only. In the small angle approximation the transverse deflection can be derived as a 2×2 jacobian matrix relating the observed source position θ_i to its true position on the transverse source plane θ'_j as $\partial\theta'_i/\partial\theta_j = \delta_{ij} + \psi_{ij}$ such that

$$\psi_{ij} = \int_0^{\chi_\infty} d\chi\ g(\chi) \left(\frac{\dot{V}_{i,j} + V_{z,ij} - V_{i,zj}}{a^2} \right) \tag{2}$$

with $g(\chi) \equiv \chi \int_\chi^{\chi_\infty} d\tilde{\chi}\,(1 - \chi/\tilde{\chi})\,W(\tilde{\chi})$ a weighted integral of the normalised source distribution function $W(\chi)$ along the line of sight.

In the case examined here, the metric perturbations V_i are sourced by vector perturbations in the cosmic string stress–energy tensor. These are described in terms of a divergenceless vector contribution to the string momentum ω_i and a divergenceless and traceless contribution to the anisotropic stress Π_i. The sources are coupled to the metric perturbation via the Einstein equations

$$\begin{aligned} V_i &= \frac{16\pi G a^2}{k^2}\omega_i \\ \dot{V}_i &= -\frac{8\pi G a^2 \Pi_i}{k} - \frac{2\dot{a}}{a}\left(\frac{16\pi G a^2}{k^2}\right)\omega_i, \end{aligned} \tag{3}$$

where the perturbations have been implicitly expanded in 3d plane waves $\exp(-i\vec{k}\cdot\vec{x})$. V_i and $\dot{V}_i$ can then be eliminated to obtain the distortion tensor ψ_{ij} in terms of the vector sources ω_i and Π_i

$$\begin{aligned} \psi_{ij} &= \frac{2G}{\pi^2}\int_0^{\chi_\infty} d\chi g(\chi) \int_{-\infty}^{\infty} d^3k e^{i\vec{k}\cdot\vec{x}} \times \\ & \hat{k}_j\left(\hat{k}_i\omega_z - \hat{k}_z\omega_i - 2\frac{\dot{a}}{a}\frac{\omega_i}{k} - \frac{1}{2}\Pi_i\right), \end{aligned} \tag{4}$$

where $\hat{k}_i \equiv k_i/|\vec{k}|$. The convergence, shear, and rotation modes can then be inferred from the distortion tensor using

$$-\psi_{ij} \equiv \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 + \rho \\ \gamma_2 - \rho & \kappa - \gamma_1 \end{pmatrix}. \tag{5}$$

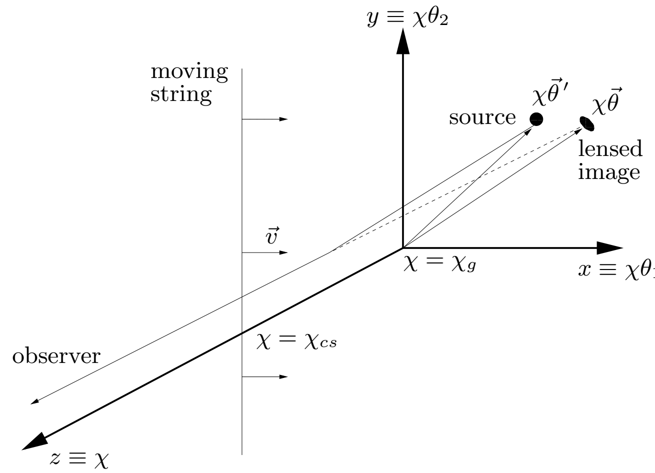

FIG. 1: The geometrical setup for the single string lensing calculation. The moving string is aligned with the y-axis, perpendicular to the line of sight. A light ray sourced at position $\chi\vec{\theta}'$ in the source plane ($\chi = \chi_g$) is imaged onto $\chi\vec{\theta}$. The vector source due to the moving string can rotate an image in addition to the usual shearing for the general case where the string is not aligned with the y-axis.

Deflection pattern around a single string. For simplicity the straight string is assumed to be aligned with the orthogonal frame of reference as shown in Fig. 1, moving with velocity v_{cs} perpendicular to the line of sight at a distance χ_{cs}. The source being lensed is placed behind the string at a distance χ_g at position $x = \chi\theta_1$, $y = \chi\theta_2$ in the orthogonal plane. The vector string velocity (or momentum) $\vec{v} = (v_{cs}, 0, 0)\delta(z - \chi_{cs})\delta(x)$ is composed of irreducible scalar and vector components v^S and $\vec{\omega}$, with $\vec{v} = \vec{\nabla}v^S + \vec{\omega}$, where $\vec{\omega}$ is divergenceless and $\vec{\nabla}\cdot\vec{v} = \nabla^2 v^S$. Thus in Fourier space the vorticity component takes the form

$$\vec{\omega} = \vec{v} - \frac{\vec{k}}{k^2}\left(\vec{k}\cdot\vec{v}\right) = 2\pi v_{cs} e^{-ik_z\chi_{cs}}\delta(k_y)(\hat{k}_z^2, 0, -\hat{k}_z\hat{k}_x)\,. \tag{6}$$

Given the geometry of the setup only the 11 component of the distortion tensor is sourced by the ω and no rotation is induced in the lensed image. The result is generalised to a string aligned in a general direction by rotating the tensor once the component ψ_{11} has been obtained. A contribution to ρ is generated upon rotation to the general configuration where the string is not aligned with the y-axis.

Substituting (6) into (4) and neglecting the time–suppressed $\frac{\dot{a}}{a}\omega$ term, and the subdominant Π term [22] gives

$$\begin{aligned} \psi_{11} &= -\frac{4Gv_{cs}}{\pi}\int_0^{\chi_g} d\chi\chi\left(1 - \frac{\chi}{\chi_g}\right) \times \\ & \int_{-\infty}^{\infty} d^3k\, e^{i\left(\vec{k}\cdot\vec{x} - k_z\chi_{cs}\right)}\delta(k_y)\hat{k}_z\hat{k}_x\,. \end{aligned} \tag{7}$$

Replacing $\hat{k}_i$ with its coordinate space equivalent ∇_i, (7)

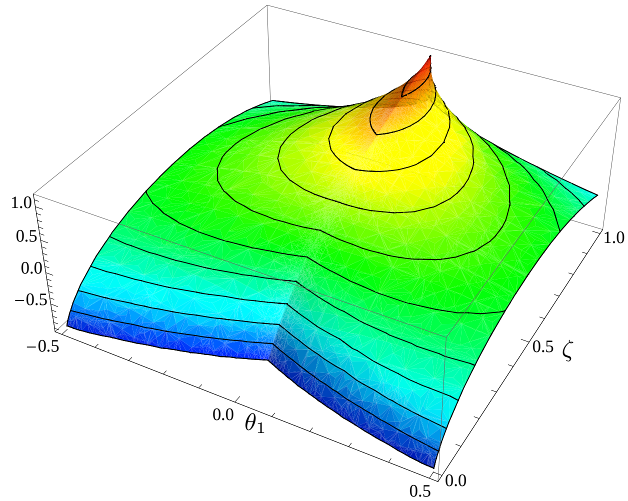

FIG. 2: The solution $\log|\rho|$, in arbitrary units, as a function of angular distance θ_1 from the string and distance of the string with respect to the distance of the source object $\zeta \equiv \chi_{cs}/\chi_g$. The solution is only valid for $\theta_1 \ll 1$. ρ is anti-symmetric about $\theta_1 = 0$.

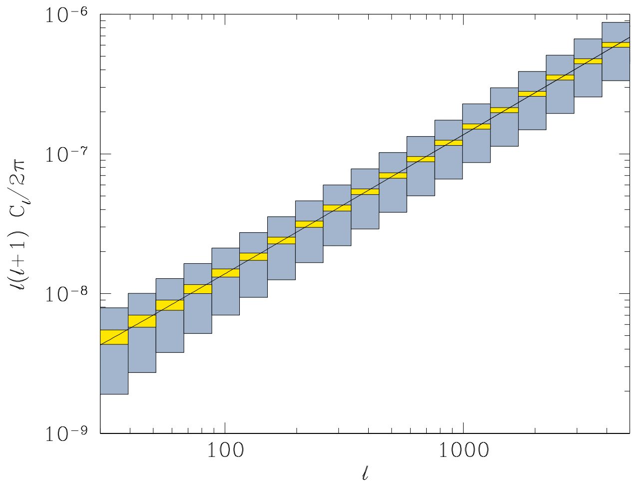

FIG. 3: The angular power spectrum of rotation for a network of strings with $G\mu = 1 \times 10^{-7}$. The blue and yellow boxes show the forecasted error for two surveys with $f_{\rm sky} = 0.1$ and $f_{\rm sky} = 0.5$ respectively. The errors include both sample and intrinsic elipticity noise contributions. The intrinsic elipticity term dominates at the relatively small scales being considered.

can be integrated by parts to obtain

$$\psi_{11} = -\frac{8Gv_{cs}}{\xi_+^2}\left\{\zeta\xi_-\left[\tan^{-1}(\frac{1}{\theta_1}) + \tan^{-1}\left(\frac{\xi_+ - \zeta}{\zeta\theta_1}\right)\right] + \theta_1\xi_+ + \frac{\zeta\theta_1}{\xi_+}\log\left(\frac{\xi_+}{\zeta^2} - \frac{2}{\zeta} + 1\right)\right\}, \quad (8)$$

with boundary terms vanishing, $\zeta \equiv \chi_{cs}/\chi_g$ and $\xi_\pm \equiv (1 \pm \theta_1^2)$. This solution maps the only non vanishing term of the distortion matrix for a single string aligned with the y-axis as a function of the string position relative to the source and the transverse angular distance from the string. It is valid in the weak lensing $\psi_{11} \ll 1$ and small angle $\theta_1 \ll 1$ regime. The rotation ρ arising from the general case where the string is moving in a direction which is *not* aligned with the θ_1 axis is obtained by rotating the solution by an angle α to give $\rho = -\psi_{11}\sin(\alpha)/2$.
The solution for $-0.5 \le \theta_1 \le 0.5$ is shown in Fig. 2. Whilst the rotation peaks in the limit $\zeta \sim 1$ with a steep drop-off in the transverse direction, it extends furthest in θ_1 when $\zeta \sim 1/2$ i.e. the case where the string is placed midway between the source and observer. For this limit the solution is approximated by

$$\rho \approx 2\pi G v_{cs}\sin\alpha\left(1 - 3\theta_1^2\right), \quad (9)$$

to second order in θ_1.
Vector power spectrum. In the presence of a network of strings the signal must be calculated in terms of power spectra averaged over the sky. In this case the signal is assumed to be generated by a scaling network of cosmic strings with tension μ with the limit $G\mu < 10^{-6}$ set by the allowed contribution to the scalar angular power spectrum of the Cosmic Microwave Background [15]. In the small angle limit the quantity of interest is the 2d power spectrum of ψ_{ij}, $\langle\psi_{ij}(\vec{l}) \quad \psi^\star_{lm}(\vec{l}')\rangle = (2\pi)^2\delta^2(\vec{l} - \vec{l}')P^\psi_{ijlm}(l)$, where $\vec{l}$ is the 2d fourier transform reciprocal of $\vec{\theta}$. The 2d power spectrum for the rotation is then

$$P_\rho(l) = \int_0^{\chi_\infty} d\chi \frac{g^2(\chi)}{\chi^3} 64\pi^2 G^2 \times \left(\frac{4\dot{a}^2\chi^2}{a^2 l^2}P_\omega(l) + \frac{P_\Pi(l)}{4} + \frac{2\dot{a}\chi}{al}P_{\Pi\omega}(l)\right), \quad (10)$$

where the power spectra for the source terms $P_\omega(l)$, $P_\Pi(l)$, and their cross–correlation $P_{\Pi\omega}(l)$ in the small angle limit ($k_z \ll l/\chi$) have been introduced.
The source spectra for scaling networks of cosmic strings can be written in terms of structure functions $P_X(k\chi, k\chi')$ which have been measured from numerical simulations [20, 23, 24] and computed from semi-analytical models [25, 26]. The unequal time correlators for the source terms are related to the structure functions through scaling laws

$$\langle\omega_i(\vec{k},\eta)\omega^\star_j(\vec{k}',\eta')\rangle = (2\pi)^3\delta^3(\vec{k} - \vec{k}')P_{ij}\frac{P_\omega(k\chi, k\chi')}{\sqrt{\chi\chi'}}, \quad (11)$$

with projector $P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$ and similar relations for $\langle\Pi\Pi^\star\rangle$ and $\langle\Pi\omega^\star\rangle$ correlations. For this case, in the small angle limit, only the diagonal of the structure functions is relevant with $P_\omega(k\chi, k\chi') \to P_\omega(l)$ where $l \approx k\chi$.
To determine whether a cosmic string network could potentially be detected, (10) can be computed numerically.

A simple power law description for the structure functions can be adopted. This is justified by causality requirements and the relative normalisations of the different correlations can be taken from the numerical results of [22, 24, 27]. An inverse scaling with l is assumed such that

$$P_\omega(l) = (G\mu)^2 \, l^{-1} \,, \tag{12}$$

with causality imposed relative normalisations $\omega : \Pi : \Pi\omega = 1 : 0.25 : 0.1$. An overall amplitude $(G\mu)^2 = 10^{-14}$ is used throughout.

Fig. 3 shows the power spectrum of the rotation C_ℓ^ρ where in the small angle limit $\ell \approx l$ and $C_\ell^\rho \sim (2\pi)^2 P_\rho(l)$. The integral in (10) is computed assuming a background galaxy distribution as a function of redshift z as $w(z) \sim z^2 \exp(-z/z_0)$ with $z_0 = 0.4$ and taking the maximum redshift to be $z = 6$. Cosmological parameters $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $h = 0.72$ are used. Expected errors for two surveys covering 10% and 50% of the sky are also shown. The errors include contributions from both sample and intrinsic elipticity noise variance although the latter dominates the errors at these scales. For both surveys a background galaxy density of 100 galaxies per square arcminute and an average intrinsic elipticity of 0.3 was assumed.

As shown a distinct weak lensing signal generated by cosmic strings is predicted: rotation with a specific power spectrum. Its intensity is below current sensitivity but provides an ideal target for projected observations. If cosmic string networks exist with $G\mu \sim 10^{-7}$ then the effect should be detectable with the next generation of surveys [3, 28]. Should it not be observed then the constraints on a string network will become considerably tighter. The only caveat is that at this level we can no longer assume that no curl–like modes are generated by lensing. Separating the corresponding systematics out in these surveys will therefore be more challenging. Yet, the distinct spectral signature of the string signal is likely to provide a simple solution to this problem.

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